

# Constraining the generalized uncertainty principle with gravitational wave

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**Abstract:** We show that the recently observed gravitational-wave signal of the event GW150914 can be used to constrain dimensionless parameter of generalized uncertainty principle (GUP). According to the Heisenberg uncertainty principle (HUP) and the event GW150914 data, we derive the standard energy-momentum dispersion relation and calculate difference between the speed of graviton and the speed of light, that is  $\Delta v$ . Subsequently, by utilizing two proposals for GUP, we generalize our work to quantum gravity case and obtain the modified speed of graviton. Finally, comparing the modified speed of graviton with  $\Delta v$ , the improved upper bound of the GUP parameters are found. The results show that the upper limit of the GUP parameters  $\beta_0$  and  $\alpha_0$  are  $2.33285 \times 10^{60}$  and  $1.80912 \times 10^{20}$ .

## 1 Introduction

Several versions of quantum gravity models is the existence of a fundamental scale of length that can be identified with the Planck scale. This view is also supposed by Gedanken experiments [1]. In this scenario, the Heisenberg uncertainty principle (HUP) can be changed to the so-called generalized uncertainty principle (GUP) which has various implications on a wide range of physical systems [2–24]. For example, effects of the GUP on the evolution of black holes were calculated in Refs. [5–14]. The GUP-corrected quantum Hall effect was investigated in Ref. [15]. The impact of the GUP on neutrino oscillations was studied in Ref. [16]. In Refs. [17, 18], with the help of GUP, the authors discussed the thermodynamics of the Friedmann-Robertson-Walker universe and the inflation preheating in cosmology. The effect of GUP also used to calculate the entropic force [19, 20]. According to the GUP, the critical temperature and the Helmholtz free energy of Bose-Einstein condensation (BEC) in the relativistic ideal Bose gas are computed in Ref. [21]. In Refs. [22, 23], the Faizal and collaborators incorporated the generalized uncertainty principle into Lifshitz field theories and showed that the breaking of supersymmetry by a non-anticommutative deformation can be used to generate the GUP. Moreover, an exciting discovery should be mentioned that the effects of GUP can be probed via the quantum optics [24]. Based on those works, one can find that there are two kinds of GUP are most studied. One was proposed by Kempf, Mangano and Mann (GUP I)

$$\Delta x \Delta p \geq \hbar \left[ 1 + \beta (\Delta p)^2 \right] / 2, \quad (1)$$

where  $\Delta x$  and  $\Delta p$  are the uncertainties for position and momentum, respectively.  $\beta = \beta_0 \ell_p^2 / \hbar^2 = \beta_0 / M_p^2 c^2$ ,  $\beta_0$  stands for a positive dimensionless parameter, which is called the GUP parameter,  $\ell_p$  represents the Planck length,  $M_p$  is the Planck mass and the Planck energy is  $M_p c^2 = 1.2 \times 10^{28} \text{eV}$ . Eq. (1) implies a nonzero minimal uncertainty  $\Delta x_{\min} \approx \ell_p \sqrt{\beta_0}$ . It should be noted that the derivation of Eq. (1) is relied on the modified fundamental commutation relation  $[x_i, p_j] = i\hbar \delta_{ij} [1 + \beta p^2]$  with the position operator  $x_i$  and the momentum operator  $p_i$  [25]. The other one was proposed in Refs. [26, 27], which admits a minimal length and a maximal momentum (GUP II)

$$\Delta x \Delta p \geq \hbar \left[ 1 - 2\alpha \Delta p + 4\alpha^2 (\Delta p)^2 \right] / 2, \quad (2)$$

where  $\alpha = \alpha_0 / M_p c = \alpha_0 \ell_p / \hbar$ , the  $\alpha_0$  is the GUP parameter. The inequality (2) is equivalent to the modified fundamental commutation relation  $[x_i, p_j] = i\hbar [\delta_{ij} - \alpha (p \delta_{ij} + p_i p_j / p) + \alpha^2 (p^2 \delta_{ij} + 3p_i p_j)]$ . From Eq. (1), one can easily obtain the minimal length  $\Delta x_{\min} \approx \alpha_0 \ell_p$  and the maximum momentum  $\Delta p_{\max} \approx \ell_p / \alpha_0$ .

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Theoretically, the GUP parameters  $\beta_0$  and  $\alpha_0$  are always assumed to be of the order of unity, in which case the corrections are negligible unless lengths or energies approach to the Planck length  $\ell_p$  or Planck energy  $E_p$ . However, if the assumptions on the GUP parameters are not taken a priori, the GUP parameters should be constrained by the current experiments [28–35]. Very recently, it was argued that the modified dispersion relations (MRD) motivated by quantum gravity that can affect the propagation of the observed gravitational-wave signal. Therefore, one can use the Laser Interferometer Gravitational-Wave Observatory (LIGO) data on event GW150914 to constrain the possibility of Lorentz violation in graviton propagation. In Ref. [36], the authors showed that the upper bound on the difference between the velocities of light and gravitational waves is  $|\Delta v| \leq 10^{-17}$ . According to the event GW150914, Arzano and Calcagni illustrated the bound on the characteristic quantum-gravity mass scale is  $M > 4 \times 10^4 \text{eV}$ , which is much weaker than that coming from photon propagation from gamma-ray bursts. Meanwhile, they found that a phenomenological dispersion relation  $\omega^2 = k^2 (1 + \alpha k^n / M^n)$  is compatible with observations and has a phenomenologically viable mass  $M > 10 \text{TeV}$  only in the quite restrictive range  $0 < n < 0.68$  [37]. In Ref. [38], in order to explain the reason why the speed of the gravitational wave  $v_g$  is smaller than that of the light  $c$ , Gwak, Kim and Lee studied the speed of the graviton in event GW150914 by using gravity’s rainbow. They pointed out that the upper bound of rainbow parameter  $\eta$  is smaller than  $4.6 \times 10^{59}$  at the frequency 250Hz, which indicates the gravity’s rainbow is very small in low energy event of GW150914.

On the other hand, it is well known that the GUP is also motivated by the quantum gravity. For this reason, inspired by previous achievements, it is meaningful to investigate the possibility of using the event GW150914 to constrain the GUP parameters  $\beta_0$  and  $\alpha_0$ . In this paper, we firstly calculate the gap between the speed of the graviton and light via the HUP. Then, according to Eq. (1) and Eq. (2), we obtained the corresponding modified dispersion relations. Finally, comparing the gap between the speed of the graviton and light  $\Delta v$  with the GUP corrected speed of the graviton, the improved upper bound of the GUP parameters are found.

The organization of the paper as follows. In section 2, we derive the standard energy-momentum dispersion relation that corresponds to HUP. Then, following the defending of the group speed of the wave front, the difference between the speed of graviton and the speed of light is obtained. In Section 3 and Section 4, we will generalize our work to quantum gravity case, the GUP effects on the speed of graviton is computed. According to the calculation of the GUP effect on the speed of graviton measurement, the improved bounds on the GUP parameters  $\beta_0$  and  $\alpha_0$  are obtained. The paper ends with the conclusion in Section 5.

## 2 The speed of graviton in the event GW150914

Let us begin by considering the conventional Heisenberg uncertainty principle  $\Delta x \Delta p \geq \hbar/2$ . This inequality is equivalent to the Heisenberg algebra  $[x_i, p_j] = i\hbar \delta_{ij}$ , and the position and momentum operators are

$$x_i = x_{0i}, \quad p_i = p_{0i}. \quad (3)$$

Following the procedure in Ref. [39], we redefine that  $p_0 = k_0$  and  $p_i = k_i$ . In the gravitational spacetime, the background metric ansatz that we will study is

$$ds^2 = g_{ab} dx^a dx^b = g_{00} c^2 dt^2 + g_{ij} dx^i dx^j. \quad (4)$$

The equation above leads to the square of the four-momentum  $p_a p^a = g_{ab} p^a p^b = g_{00} (k^0)^2 + g_{ij} p^i p^j = g_{00} (k^0)^2 + g_{ij} k^i k^j$ . Considering the square of the momentum  $p_i$  in this background satisfy the relation  $p^2 = p^i p_i = g_{ij} k^i k^j$ , the square of the four-momentum can be rewritten as

$$p_a p^a = g_{00} (k^0)^2 + p^2. \quad (5)$$

It should be mentioned that the right hand in RHS of Eq. (5) form the original dispersion relation  $k_a k^a = g_{00} (k^0)^2 + p^2 = -m^2 c^2$ . Therefore, Eq. (5) takes the form

$$p_a p^a = -m^2 c^2, \quad (6)$$

and the time component of the momentum is given by

$$(p^0)^2 = (-p^2 - m^2 c^2)/g_{00}. \quad (7)$$

It is well known that the energy of a particle can be defined as the following form

$$\omega/c = -\xi_a p^a = -g_{ab} \xi^a p^b, \quad (8)$$

where  $\xi^a = (1, 0, 0, \dots)$  is the Killing vector. With the help of Eq. (8), the energy in the metric (4) can be defended as [39]

$$\omega = -g_{00} c p^0. \quad (9)$$

Now, substituting Eq. (9) into Eq. (7), one can express the energy of a particle in terms of high energy momentum and the mass

$$\omega^2 = (-g_{00} c p^0)^2 = -g_{00} (p^2 c^2 + m^2 c^4). \quad (10)$$

Since the LIGO is in a weak gravitational spacetime, here we only work in the Minkowski spacetime, that is  $g_{00} = -1$ . Thus, Eq. (10) reduces to the standard energy-momentum dispersion relation  $\omega^2 = p^2 c^2 + m^2 c^4$ . Meanwhile, if assuming that the GWs propagate as free waves, one can calculate the speed of graviton by using the group speed of the wave front  $v := \partial\omega/\partial p$ , where  $\omega$  and  $p$  represent the energy and the momentum, respectively [40, 41]. For the standard energy-momentum dispersion relation, the velocity of graviton is given by

$$v_g \approx c (1 - m_g^2 c^4 / 2\omega_g^2), \quad (11)$$

where  $\omega_g$  and  $m_g$  are the energy and mass of graviton, respectively. Considering  $h = 4.136 \times 10^{-15} \text{eV} \cdot \text{s}$  and  $c = 3 \times 10^8 \text{m/s}$ , the fractional deviation of the wave propagation velocity from that of light is equal to  $\Delta v = c - v_g = m_g^2 c^5 / 2\omega_g^2$ . In Refs. [42, 43], the authors showed that the peak of the signal of GW150914 appears at 150Hz. So that, the maximum energy is  $\omega_g \approx 6.024 \times 10^{-13} \text{eV}$ . Meanwhile, by analyzing the arrival time of difference frequency components of signal, the upper bound for the mass of the graviton is given by  $m_g \leq 1.2 \times 10^{-22} \text{eV}/c^2$ . Therefore, one has

$$\Delta v < 5.6119 \times 10^{-12} \text{m/s}. \quad (12)$$

It is clear that the difference between the speed of graviton  $v_g$  and the speed of light  $c$  is very small. However, with the help of Eq. (12), one can investigate the proper range of the parameter in GUP motivated by quantum gravity. In the next section, the upper limit of GUP parameter will be calculated by taking into account the gap between the GUP corrected velocity of graviton that obtained from event GW150914 and Eq. (12).

### 3 Bounds on GUP parameters $\beta_0$

In this section, we will set the upper bound of GUP parameter  $\beta_0$  by taking into account the speed that obtained from event GW150914 and Eq. (12). In Eq. (1), the operators of position  $x_i$  and momentum  $p_i$  can be defended as

$$x_i = x_{0i}, \quad p_0 = k_0, \quad p_i = p_{0i} (1 + \beta p^2) = k_i (1 + \beta k^2), \quad (13)$$

In the Minkowski spacetime, the modified square of the four-momentum is given by

$$p_a p^a = g_{ab} p^a p^b = -(p^{i0})^2 + g_{ij} p^i p^j = -(k^0)^2 + g_{ij} k^i k^j (1 + \beta k^2)^2. \quad (14)$$

Then, retaining terms up to  $\mathcal{O}(\beta^2)$ , the above equation is rewritten as

$$p_a p^a = -(k^0)^2 + k^2 + 2\beta k^2 k^2, \quad (15)$$

where we use the relations  $k^2 = g_{ij} k^i k^j$  and  $k = \sqrt{g_{ij} k^i k^j}$ . It should be noted that the first two terms of Eq. (15) form the usual dispersion relation. Therefore, Eq. (15) would be  $k_a k^a = -(k^0)^2 + k^2 = -m^2 c^2$ , and the time component of the momentum becomes

$$(p^0)^2 = m^2 c^2 + p^2 (1 - 2\beta p^2). \quad (16)$$

According to Eq. (9), the energy of a particle can be expressed in terms of the momentum and the mass as follows

$$\omega^2 = m^2 c^4 + p^2 c^2 (1 - 2\beta p^2). \quad (17)$$

Note that the  $\beta \rightarrow 0$ , one gets back the results of the standard energy-momentum dispersion relation. Next, using Eq. (11), the group velocity of a massless graviton with a modified dispersion relation (17) is approximately equal to

$$v_{massless} = \frac{\partial \omega}{\partial p} = \frac{c(1 - 4p^2\beta)}{\sqrt{1 - 2p^2\beta}} \approx c(1 - 3\beta p^2). \quad (18)$$

In the infrared, one has a correction to the massless dispersion relation  $\omega^2 = p^2 c^2$ . So that, the Eq. (18) can be rewritten as

$$v_{massless} \approx c(1 - 3\beta \omega^2 / c^2). \quad (19)$$

The difference between the modified speed of the graviton and the speed of light is

$$\Delta v = c - v_{massless} = 3\beta \omega^2 / c = 3\beta_0 \omega^2 / M_p^2 c^3. \quad (20)$$

It should be noted that, in the previous work [20], one assumes that  $\beta_0 \sim 1$ . However, if this assumption is made, it will lead to a non-zero, but virtually unmeasurable effect of GUP. Conversely, without this assumption, we can set the improved bound of GUP parameter by comparing  $\Delta v$  with Eq. (12), that is,  $3\beta_0 \omega^2 / M_p^2 c^3 < 5.6119 \times 10^{-12} \text{m/s}$ . Therefore, the upper bound of GUP parameter is given by

$$\beta_0 < 2.33285 \times 10^{60}. \quad (21)$$

Comparing this bound with those results in Table. 1, it is found that our result is weaker than those set by the tunneling current in a scanning tunneling microscope, the position measurement, the Hydrogen lamb shift, the  $^{87}\text{Rb}$  cold-atom-recoil experiment and the Landau levels, whereas it is more stringent than ones derived in previous work [33]. Besides, in the previous work [20], we set  $\beta_0 \sim 1$ , which is in accordance Eq. (21).

Table 1: Current experimental bounds on GUP parameter  $\beta_0$

measurement/experiment	$\beta_0$	Refs.
Electron tunneling	$10^{21}$	[28, 29]
Position measurement	$10^{34}$	[28, 29]
Hydrogen Lamb shift	$10^{36}$	[28, 29]
$^{87}\text{Rb}$ cold-atom-recoil experiment	$10^{39}$	[30]
Landau levels	$10^{50}$	[28, 29]
Light deflection(Solar system data)	$10^{69}$	[33]
Light deflection(Pulsar PRS B 1913+16 data)	$10^{71}$	[33]
Modified mass-temperature relation	$10^{78}$	[33]
Light deflection	$10^{78}$	[33]

## 4 Bounds on GUP parameters $\alpha_0$

In this section we apply the above formalism to the GUP II case, the effect of the GUP-induced term can be measurable. Now, let us define the operators of position  $x_i$  and momentum  $p_i$  of GUP II as follows

$$x_i = x_{0i}, \quad p_0 = k_0, \quad p_i = p_{0i} (1 - \alpha p + 2\alpha^2 p^2) = k_i (1 - \alpha k + 2\alpha^2 k^2), \quad (22)$$

Similar to the case of GUP I, the modified dispersion relation is derived as [44]

$$\omega^2 = m^2 c^4 + p^2 c^2 (1 - \alpha p)^2. \quad (23)$$

Using Eq. (8) and the massless dispersion relation, and then, ignoring the higher order term of  $\mathcal{O}(\alpha)$ , the group velocity of a massless graviton is  $\tilde{v}_{massless} = c(1 - 2\alpha\omega/c)$ . Therefore, one has

$$\Delta v = c - \tilde{v}_{massless} = 2\alpha\omega < 5.6119 \times 10^{-12} \text{m/s}. \quad (24)$$

From above inequality, the upper bound of GUP parameter is given by  $\alpha < 4.52281$  at the frequency 150Hz, it will put the upper bound of GUP parameter

$$\alpha_0 < 1.80912 \times 10^{20}. \quad (25)$$

It is clear the bound on  $\alpha_0$  is 40 orders better than the one on  $\beta_0$ , it indicates that the effect of minimal length and maximal momentum is large than the effect of minimal length. This feature makes GUP II case easier to test than the GUP I case. In Table. 2, one can see the bound on  $\alpha_0$  set by the high-energy physics experiments and the low-energy physics experiments. Compared to those results in Table. 2, it is found that

Table 2: Current experimental bounds on GUP parameter  $\alpha_0$

measurement/experiment	$\alpha_0$	Refs.
Anomalous magnetic moment of the muon	$10^8$	[15]
Hydrogen Lamb shift	$10^{10}$	[30]
Electron tunneling	$10^{11}$	[30]
$^{87}\text{Rb}$ cold-atom-recoil experiment	$10^{14}$	[34]
Position measurement	$10^{17}$	[30]
Superconductivity	$10^{17}$	[15]
Landau levels	$10^{23}$	[30]

the improved upper bound in Eq. (25) is far weaker than that set by anomalous magnetic moment of the muon, the electron tunneling, the Hydrogen Lamb shift, the  $^{87}\text{Rb}$  cold-atom-recoil experiment, the position measurement and superconductivity experiment while it is stringent than Landau levels. Moreover, one can set  $\alpha_0 \sim 1$  since it is contradict with Eq. (25).

## 5 Conclusion

In the previous work, it is normally assume the GUP parameters are of the order of unity. However, such an assumption would lead to the correction too small to be measured. On the other hand, if this assumption is not made, one can constrain the GUP parameters by the current experiments. In this paper, we have discussed two proposals for the GUP and managed to use the results from the event GW150914 to calculate the modified speed of the graviton. Finally, we set the improved upper bound on the GUP parameters  $\beta_0$  and  $\alpha_0$ . The results showed that the modified speeds of the gravitons are related to the effect of GUP. When the GUP parameters approach to zero, the modifications reduce to the original cases. Furthermore, a  $10^{60}$ -level bound on  $\beta_0$  and  $10^{20}$ -level bound on  $\alpha_0$  are obtained. We found the bound on  $\alpha_0$  is 40 orders better than the one on  $\beta_0$ , it indicates that the effect of minimal length and maximal momentum is large than the effect of minimal length. This feature makes GUP II case easier to test than the GUP I case. Meanwhile, from Eqs. (22) and (25), it is shown that the improved GUP parameters are associated with the energy  $\omega_g$ , the mass of graviton  $m_g$ , Planck mass  $M_p$  and the speed of light  $c$  while the bounds set from others experiments, such as anomalous magnetic moment of the muon is only related to  $m^2/M_p^2$ , hence this case our bounds are more weak. It would also be interesting to note that a lot of work predict the pulsars (spinning neutron stars) are promising candidates for producing the gravitational wave signals [45–47]. If those theories are proved by advanced LIGO and advanced Virgo, one can obtain more accurate measurements in the future.

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